A 2-D Finite-Element Model for Electro-Thermal Transients in Accelerator Magnets

L. Bortot[1], B. Auchmann[1,2], I. Cortes Garcia[4], A.M. Fernandez Navarro[1], M. Maciejewski[1,3], M. Prioli[1], S. Schöps[4],and A.P. Verweij[1]

[1] CERN, Geneva, Switzerland, E-mail: lorenzo.bortot@cern.ch

[2] Paul Scherrer Institut, Villigen, Switzerland

[3] Łódź University of Technology, Łódź, Poland

[4] Technische Universität Darmstadt, Darmstadt, Germany

Superconducting accelerator magnets require sophisticated monitoring and means of protection due to the high energy stored in the magnetic field. Numerical simulations play a crucial role in understanding transient phenomena occurring within the magnet, and could therefore help in preventing disruptive consequences. We present a 2-D FEM model for the simulation of the electro-thermal transients occurring in superconducting accelerator magnets. The magnetoquasistatic problem is solved with a modified magnetic vector potential formulation, where the cable eddy currents are resolved in terms of their equivalent magnetization. The heat balance equation is then investigated, and the relevant heat sources are discussed. The model implements a two-port component interface and is resolved as part of an electrical circuit, in a cooperative simulation scheme with a lumped-parameter network.

***Index Terms*—Accelerator Magnet, Quench, Finite-Element Analysis, Equivalent Magnetization, Eddy Currents, Large Hadron Collider.**

I. Introduction

Circular accelerators for high-energy particle physics require intense magnetic fields to control the trajectories of particle beams. These fields are generated by means of high-energy superconducting magnets, which are electrically connected in series, and operated in circuits that contain up to hundreds of elements. It is of paramount importance to ensure a safe management of the stored energy which, if released in an uncontrolled way, could compromise the integrity of the superconducting circuit. This is critical in case of an event such as a quench, where the energy is released as Ohmic losses. Quenches cannot always be prevented, but must be considered among the possible operational scenarios. Generally, dedicated quench detection and protection systems are in place to quickly discharge the stored energy, to avoid overheating of the coil. These systems influence both the quench evolution in the coil and the electrical transient in the rest of the circuit. A careful analysis of the electro-thermal transient is fundamental for the design of both the magnet and the quench protection system, and for the safe operation of the circuit.

We present a coupled electro-thermal 2-D model of a superconducting magnet. The model accounts for the non-linear temperature- and field-dependent material properties and for the induced eddy-currents in the cable, as well as in the coil's copper wedges. The construction of the magnet cross-section is realized considering the coil composed of single half-turns, each one treated as a fundamental domain over which material properties and physics laws are homogenized. The model has been developed as a modular component of a wider numerical architecture, which implements the concept of cooperative simulation. The aim of the architecture is to resolve the electrodynamic coupling between the magnet, the protection systems, and the remaining network, leading to consistent simulations.

II. Method

The electro-thermal model combines the magnetic vector potential formulation $\nabla \times \vec{A} = \vec{B}$ with the heat balance equation. The magnetoquasistatic field solution, fixed with a Coulomb gauge and driven by the current density source $\vec{J}_\mathrm{s}$, determines the magnet's electrodynamics and the related thermal losses. The cable eddy currents' term $\sigma\partial_t\vec{A}$, proportional to the conductivity σ, is replaced by an equivalent magnetization factor $\vec{M}_\mathrm{cc}$ [1], [2] proportional to the time derivative of the magnetic flux density $\partial_t\vec{B}$ via an equivalent time constant τ_loop. The cable's persistent magnetization $\vec{M}_\mathrm{pers}$ [1], [3] is included as $\vec{M} = \vec{M}_\mathrm{cc}+\vec{M}_\mathrm{pers}$, leading the dynamic effects to be consistently included in the constitutive law $\vec{B} = \nu^{-1}(\vec{H}+\vec{M})$, where $\vec{H}$ represent the magnetic field and ν^{-1} is the magnetic permeability. The temperature field T is determined by the balance of the heat sources Q with the heat $\rho C_\mathrm{p}\partial_t T$ stored in the system and the heat flux $\nabla \cdot \vec{q}$. The proposed formulation leads on the 2-D domain Ω the following formal set of equations, where $\vec{A} = (0, 0, A_z)$

$$\begin{cases} \vec{M}_\mathrm{cc} = -\nu\tau_\mathrm{loop}\partial_t\vec{B} \\ \nabla \times (\nu\ \nabla \times \vec{A}) = \vec{J}_\mathrm{s} + \sigma\partial_t\vec{A} + \nabla \times \vec{M}. \\ \rho C_\mathrm{p}\partial_t T + \nabla \cdot \vec{q} = Q \end{cases} \tag{1}$$

The boundary $\partial\Omega$ is linked to Ω through a layer of infinite domain elements [4]. Let $\vec{n}$ be the outward pointing vector: if no symmetry is exploited, the Dirichlet boundary condition $\vec{n} \times \vec{A} = 0$ is imposed. The formulation is implemented in the FEM model at the scale of half-turns (ht), whereas a turn (tu) is composed by two related half-turns forming a closed loop, as in Fig. 1). The model is driven by an external current I_s in order to be coupled with the the co-simulation algorithm. The current is distributed over the n_ht half-turns as $\vec{J}_\mathrm{s} = \vec{\chi} I_\mathrm{s}$ [5], through the density function $\vec{\chi} = \sum_{i=1}^{n_\mathrm{ht}} \vec{\chi}_{\mathrm{ht},i}$.

A. Electrodynamics and Heat Balance

The induced eddy currents in a fully transposed superconducting cable can be separated into the contributions of the inter-filament (IFCC) and the inter-strand (ISCC) coupling currents. They are both included in the model through their equivalent magnetization representation; see Eq. 1 and Fig. 2. This avoids to resolve the coupling currents' paths, which would require an explicit discretization of the turn's domain at the micrometric scale of the superconducting filaments. The magnet's structural elements host parasitic eddy currents, negligible for the laminated iron yoke, although relevant for the copper wedges in the coil assembly (see Fig. 1). It is worth noting that wedges are individually insulated and do not form a loop across the aperture. For this reason, a weak constraint of zero net current is added to each wedge domain.

The generic half-turn Ω_{ht} is considered as homogeneous. The definition of the equivalent thermal capacity $\rho C_{\text{p},\Omega_{\text{ht}}}$ accounts for the superconducting material, the copper stabilizer, the external insulation wrapping and the coolant in the cable porosity. The thermal diffusivity of the copper at cryogenic temperature ($> 10\ \text{mm}^2/\text{ms}$), permits to consider Ω_{ht} as an isothermal domain, over which the material properties are calculated from the average temperature T_{ht}. The heat diffusion term is formulated following the Fourier Law $\vec{q} = -k\nabla T$ and it accounts for the heat exchanged across the insulation layer, between turns and from turns to wedges. Due to the aspect ratio of the insulation layer (> 100 on the turn's wide side), k is symmetric and anisotropic, to neglect the tangential component of the heat flux. Three phenomena occur in the determination of the heat source term: cable coupling currents Q_{M}, eddy currents Q_{eddy} in the wedges and, in case of a quench, Joule losses Q_{Joule} in the superconductor. The Ohmic contribution $Q_{\text{Joule}} = \vec{J}_{\text{s}} \cdot \sigma_{\text{ht}}^{-1} \vec{J}_s$ is active only if the working point of the half-turn crosses the superconducting material's critical surface, causing a quench. The current sharing regime [6] is numerically reformulated using a generalized logistic function [7]: The function is suitable to fit the $E = f(J)$ characteristic of the superconducting cable [3], ensuring a finite amount of energy.

B. Interface

The model provides as feedback two quantities, per coil's unit length: the total resistance R_c and the inductive voltage U_c, calculated as the time derivative of the coil's linked flux. If the averages of the vector potential $\vec{A}_{\text{ht}}$ and the resistivity σ_{ht}^{-1} over a generic half-turn Ω_{ht} are introduced, the coil's equivalent parameters read

$$R_{\text{c}} = \sum_{i=1}^{n_{\text{ht}}} (\sigma_{\text{ht},i}^{-1}\ \Omega_{\text{ht},i}), \quad U_c = \vec{\chi} \cdot \sum_{i=1}^{n_{\text{ht}}} \partial_t \vec{A}_{\text{ht},i}. \qquad (2)$$

Once the turns' electrical connection order is defined, the definition of the contributions of resistive and inductive voltage per turn allows to evaluate the voltage-to-ground distribution along the magnet's coil, as a cumulative sum. This is of interest due to the unbalance of the two voltage contributions occurring during a quench, that could lead to unacceptable voltages to ground.

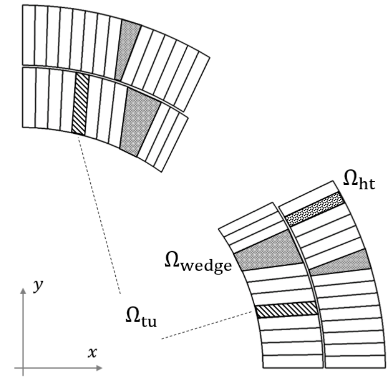

Fig. 1. Cross section of the first quadrant of a the quadrupole magnet's coil. The shaded domains Ω_{wedge} represent the structural copper wedges.

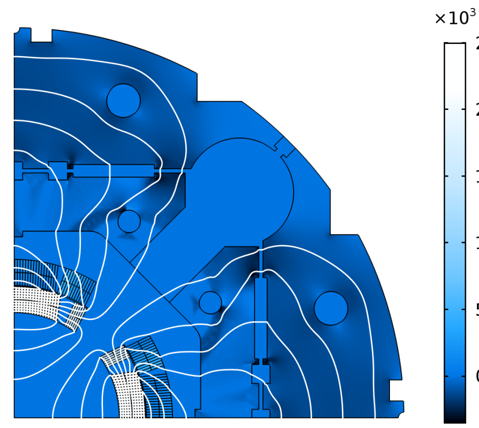

Fig. 2. IFCC and ISCC equivalent magnetization at 5 kA, during a linear ramp-up of 100 A/s, expressed in A/m as a difference of two solutions.

III. Outlook

The main achievement of this work is a consistent FEM model implemented with the COMSOL Multiphysics® software [8]: we simulated the most relevant electro-thermal transient phenomena occurring in accelerator magnets. In the full paper, together with the detailed numerical formulation, we will present additional numerical results to highlight the capabilities of our approach.

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